ChE-304 Problem Set 8

Week 9

Problem 1

You want to determine the distribution of corn yields across farms for biofuel production using a Monte Carlo analysis. There are two sources of variation for crop yields from year to year. First, the amount of fertilizer varies, which causes the yield to vary but this yield can also be influenced by random growth factors. The yield is calculated as:

$Yield = F \times a$

where F is the amount of feritlizer used (in kg/acre) and a is a growth parameter (in ton corn per kg fertilizer used).

Both the amount of fertilizer yield and the growth parameter vary according to a Cauchy distribution:

Probability distribution function:

$$P(z; z_0, \gamma) = \frac{1}{\pi\gamma}\left[\frac{\gamma^2}{(z-z_0)^2+\gamma^2}\right]$$

Cumulative distribution function:

$$CDF(z; z_0, \gamma) = \frac{1}{\pi}\arctan\left[\frac{(z-z_0)}{\gamma}\right] + \frac{1}{2}$$

Where:

z_0: is the average value of z

γ: is the half width at half maximum of the distribution (sort of like a standard deviation for a normal distribution.

Here:

$z_{0,F} = 1 kg/acre$ $\quad$ $z_{0,a} = 5\ ton\ corn/kg$

$\gamma_F = 0.1\ kg/acre$ $\quad$ $\gamma_a = 1\ ton\ corn/kg$

Use a 6-faced dice as a random number generator and plot the final distribution function. I have prepared a template to bin your results.

Hint: divide the dice result by 7 in order to always obtain a number between 0 and 1. I recommend performing at least 20 dice rolls for each parameter to get a decent result…

Solution:

From the dice roll we divide by 7 to obtain a random number between 0 and 1 (we refer to this number as RAND). Therefore, we have:

$$RAND = CDF(z; z_0, \gamma) = \frac{1}{\pi}\arctan\left[\frac{(z - z_0)}{\gamma}\right] + \frac{1}{2}$$

and we want *z*:

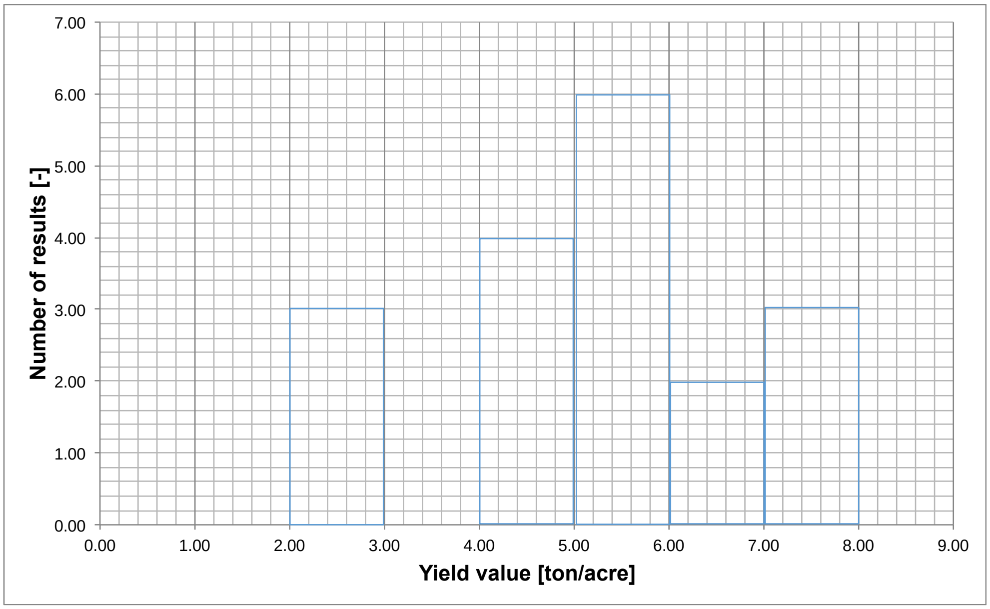
$$\left[\pi\left(RAND - \frac{1}{2}\right)\right] = \arctan\left[\frac{(z - z_0)}{\gamma}\right] \rightarrow z = z_0 + \gamma\tan\left[\pi\left(RAND - \frac{1}{2}\right)\right]$$

Therefore, for each roll of the dice we perform the following calculation:

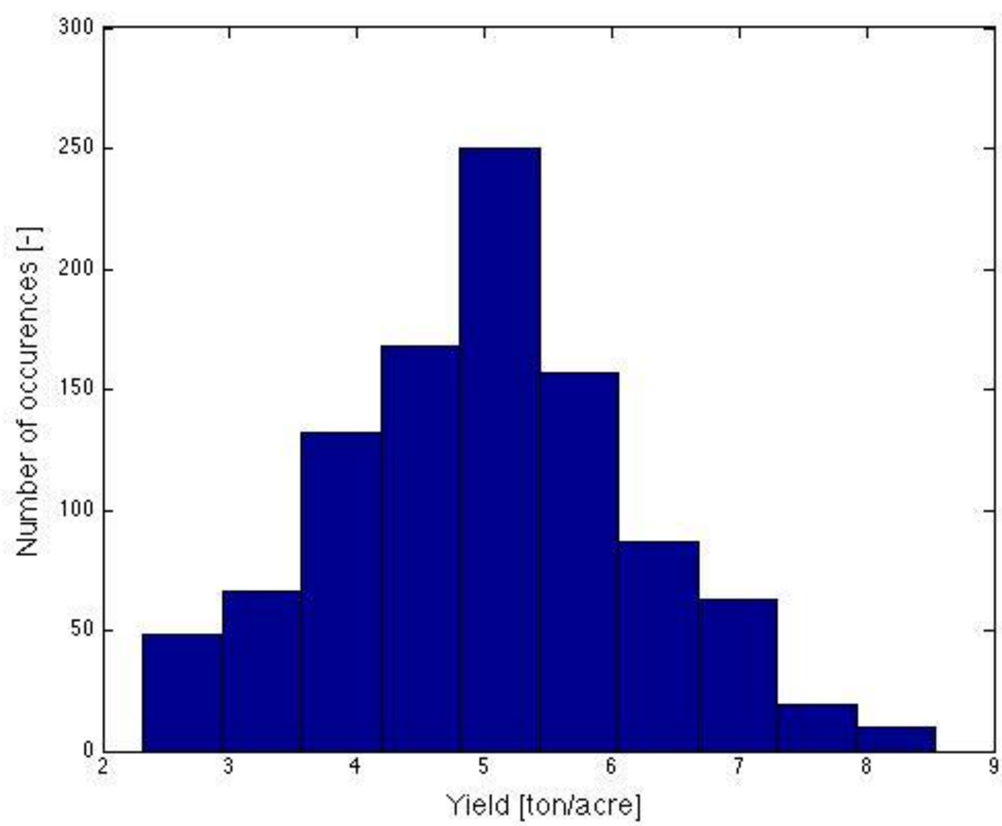
$$Yield_k = \left(z_{0,F} + \gamma_F\tan\left[\pi\left(RAND_1 - \frac{1}{2}\right)\right]\right)\left(z_{0,a} + \gamma_a\tan\left[\pi\left(RAND_2 - \frac{1}{2}\right)\right]\right)$$

Dice Roll	Dice roll result 1	RAND	Fertilizer used	Dice roll result 2	RAND	Growth parameter	Yield
1.00	4.00	0.57	1.02	2.00	0.29	4.20	**4.30**
2.00	2.00	0.29	0.92	3.00	0.43	4.77	**4.39**
3.00	4.00	0.57	1.02	4.00	0.57	5.23	**5.35**
4.00	1.00	0.14	0.79	1.00	0.14	2.92	**2.32**
5.00	2.00	0.29	0.92	3.00	0.43	4.77	**4.39**
6.00	3.00	0.43	0.98	1.00	0.14	2.92	**2.86**
7.00	5.00	0.71	1.08	6.00	0.86	7.08	**7.64**
8.00	3.00	0.43	0.98	5.00	0.71	5.80	**5.67**
9.00	5.00	0.71	1.08	6.00	0.86	7.08	**7.64**
10.00	6.00	0.86	1.21	5.00	0.71	5.80	**7.00**
11.00	3.00	0.43	0.98	3.00	0.43	4.77	**4.66**
12.00	1.00	0.14	0.79	6.00	0.86	7.08	**5.61**
13.00	2.00	0.29	0.92	4.00	0.57	5.23	**4.81**
14.00	6.00	0.86	1.21	4.00	0.57	5.23	**6.31**
15.00	6.00	0.86	1.21	3.00	0.43	4.77	**5.76**
16.00	5.00	0.71	1.08	4.00	0.57	5.23	**5.65**
17.00	5.00	0.71	1.08	5.00	0.71	5.80	**6.26**
18.00	2.00	0.29	0.92	5.00	0.71	5.80	**5.34**
19.00	1.00	0.14	0.79	1.00	0.14	2.92	**2.32**
20.00	3.00	0.43	0.98	5.00	0.71	5.80	**5.67**

If we bin the results between 0-1, 1-2, etc. and plot the resulting distribution, we have:



Note that if I perform 10’000 simulated dice rolls I get a much smoother (but similar) result:



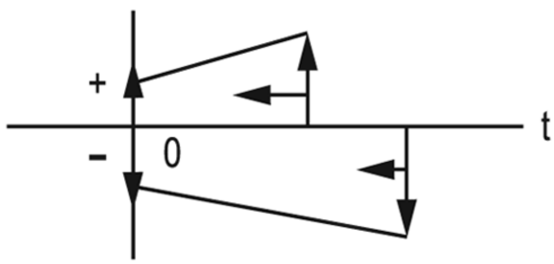
Problem 2

Someone proposes you a PV system that has a lifetime of 20 years for your house at a price of 25’000 CHF with the promise that you can continuously sell electricity at a garanteed cost that will bring 3’000 CHF in lump sum payment at the end of each year (in today’s CHF). On the other hand, you could put this money in a bank that offers a real (inflation free) interest rate of 2% (compounded once a year).

What is the net present value of you investment given this cost of money (2%)? Is this investment worth it?

Can you calculate the actual discounted rate of return of the PV investement (this might require some iteration)?

Solution:
Let’s calualte the net present value of our investment using a 5% interest rate. If it’s more than the To do so, we calculate the net present value of our full cash flow:



Here the positives are 3000 CHF/year for 20 years and the negatives are -25’000 CHF at year zero.

We can use an interest rate of 2% and see if the total is positive:

$$P_T = -25'000 + \sum_{n=1}^{20} \frac{3'000}{(1+i_d)^n} = -25'000 + \sum_{n=1}^{20} \frac{3'000}{(1+0.02)^n} = -25'000 + 49'054$$
$$= 24'054\ CHF$$

The question of whether this is a good investment is actually a bit complicated.

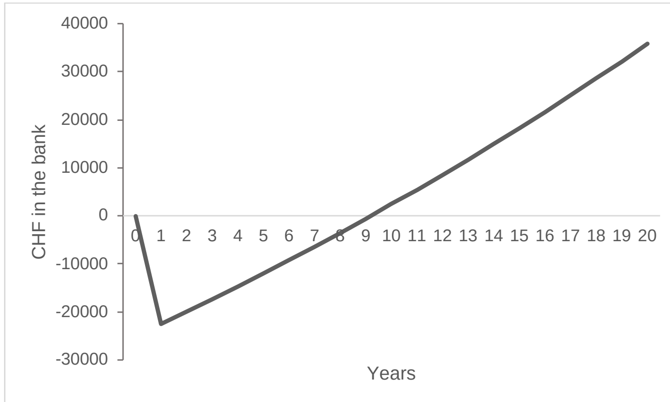
The net present value suggests that the amount is lower than 25’000 CHF, which suggests that the investment worth less than you money now. If you were to calculate the value of 24’054 CHF in 20 years you would get:

$$P\,(1+i_d)^T = 24'054(1.02)^{20} = 35'743\ CHF$$

Which is less than if you put money in the bank:

$$F = P\,(1+i_d)^T = 25'000(1.02)^{20} = 37'149\ CHF$$

However, this calculation assumes that you carry a negative forward in your Bank account and would pay negative interest on that amount:



This is normal because industry projects are usually financed with debt. However, if you had the money then there would be no negative interest and you could just carry forward the earnings made. In this case, after 20 years you would have:

$$F = \sum_{n=0}^{19} 3'000(1+i_d)^n = 72'892\ CHF$$

Note that we sum from 19 to zero because the first 3000 CHF will be received at the end of 19 years and so will have 19 years to grow and the last installement will be received at the end of 20 years and so will have zero years to grow.

Therefore, if you have the money, it's a worthwhile investment. If you have to finance it, it's not worth it.

We can play around with the value of i_d until until we get close to zero in the net present value:

$$P_T = -25'000 + \sum_{n=1}^{20} \frac{3'000}{(1+i_d)^n}$$

which gives around 10.32% for the discounted rate of return of the PV investment.

Problem 3

A nuclear power plant must provide enough money to decommission its setup at the end of its 30 year lifetime. The cost (in today's dollars, i.e. corrected for inflation) is 300 million dollars.

Assuming it uses a fund that has a real interest rate (inflation free) of 5% a year (compounds continuously), how much must the power plant owners put aside every year?

How much would they need to put aside if a lump payment at the installation of the plant was required?

Solution:

$$\int_0^{30} \bar{A}\, e^{-0.05t}\, dt = 300\, e^{-0.05(30)}\,.$$

Carrying out the integration gives:

$$\frac{\bar{A}}{0.05}\left(1 - e^{-0.05(30)}\right) = 300\, e^{-1.5}$$

or

$$\bar{A} = \frac{(300)(0.05)}{\left(e^{1.5} - 1\right)} = \text{ \$4.31 million per year,}$$

which is only a few percent of the annual carrying charge rate on a plant costing on the order of $2 billion.

We merely need to compute the present worth of $300 million at 5% per year for 30 years:

$$P = 300 \cdot e^{-0.05(30)}$$
$$= \text{\$66.94 million (in today's, i.e., constant, dollars).}$$